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Self-enforcing Employment Contracts and Business Cycle Fluctuations *

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Résumé:
On dit souvent que des relations d'emploi impliquant un partage de risque donnent une meilleure description des relations de travail que le cadre Walrasien traditionnel. De telles relations ont été introduites dans des modèles macroéconomiques sous l'hypothèse de plein engagement. Toutefois, en l'absense d'engagement, ces relations doivent être auto-exécutoires. Ce papier étudie l'impact de l'engagement limité sur les fluctuations du marché de l'emploi. Il est montré que les contrats de travail auto-exécutoires peuvent expliquer plusieurs faits caractéristiques reliés aux comouvements du salaire réel et des heures travaillées. De plus, des implications pour l'estimation de l'élasticité intertemporelle de l'offre de travail sont mises en évidence. En particulier, il est montré que les études empiriques qui négligent l'existence de contrats auto-exécutoires peuvent sous-estimer sévèrement la volonté des travailleurs de substituter les loisirs entre les périodes.

Abstract:
It is often argued that risk-sharing employment relationships provide a better description of labour relations than the standard Walrasian framework. Such relationships have been introduced in macroeconomic models under an assumption of full commitment. In the absence of commitment, however, these relationships must be self-enforcing. This paper examines the impact of limited commitment on labour market fluctuations. It is shown that self-enforcing employment contracts can explain several stylized facts related to the comovements of real wages and hours worked. Moreover, implications for the estimation of intertemporal labour supply elasticities are highlighted. In particular, it is shown that empirical studies of labour supply that neglect the existence of self-enforcing contracts may severely underestimate workers' willingness to substitute leisure intertemporally.

Keywords:
Commitment; implicit labour contracts; business cycles.

JEL classification: E3; J3

1. Introduction.

U.S. labour market fluctuations are characterized by large procyclical variations in hours worked without any systematic movements in real wages. Accounting for this stylized fact within a purely Walrasian labour market is difficult because most empirical studies report small estimates of labour supply elasticities.[1] Danthine and Donaldson (1992), Boldrin and Horvath (1995), and Gomme and Greenwood (1995) address this issue by introducing risk-sharing employment relationships into standard macroeconomic models. However, while they represent clear improvements upon the Walrasian framework, their models are based on an assumption that firms and workers are fully committed to employment relationships. This assumption is questionable, in this context, since it is relatively easy for workers or employers to end relationships if better opportunities arise.[2] Given this potential for separation, relationships will only continue if they are self-enforcing. That is, if they always give to both parties more than what outside alternatives offer. The goal of this paper is to examine if requiring that labour contracts be self-enforcing has a significant impact on the outcomes of these models. The empirical evidence presented in Beaudry and DiNardo (1991, 1995) suggests that this is a relevant issue.

The possibility that employment contracts provide workers with some degree of insurance against income fluctuations has several implications for the study of labour market fluctuations. These are discussed in Rosen (1985). First, the provision of insurance increases the sensitivity of labour supply to changes in productivity because it eliminates the income effect associated with such changes.[3] Second, the insurance component embodied in workers' earnings dissociates the behaviour of real wages from that of the underlying marginal product of labour. Hence, on the one hand, movements in hours worked need not be associated with large movements in wages even if workers' intertemporal labour supply elasticities are small.[4] On the other hand, as most empirical studies of labour supply assume that real wages measure the marginal product of labour, estimates of labour supply elasticities reported therein may suffer from a downward bias if risk-sharing is an important aspect of labour relations.

The provision of insurance is only partial when firms and workers are unable to credibly commit not to terminate relationships. Therefore, changes in the productivity of labour

[1] Pencavel (1986) reviews several microeconomic studies of labour supply. Estimates of male labour supply elasticities reported therein cluster around 0.2.

[2] Bansak and Raphael (1998) report that, in the U.S., 20% of employment relationships end within their first year of existence.

[3] This statement presupposes that perfect risk-sharing can be attained. It is the case in Danthine and Donaldson (1992), Boldrin and Horvath (1995), and Gomme and Greenwood (1995).

[4] Throughout the paper, the term "intertemporal labour supply elasticity" refers to the elasticity of labour supply, holding the marginal utility of wealth constant.

still have income effects in this case. The behaviour of wages and hours will thus differ from that described in Rosen (1985), Danthine and Donaldson (1992), Boldrin and Horvath (1995), and Gomme and Greenwood (1995).[5] It will be similar to that described in Thomas and Worrall (1988) and Beaudry and DiNardo (1995). The present paper models employment relationships by drawing on their work. A model is developed in which the job market is essentially a market for implicit contracts. Employers compete in this market by offering risk-sharing employment relationships. Participation in the job market is limited to unemployed workers and to employers with job vacancies. However, as the continuation of relationships is assumed unenforceable, employers and employed workers can terminate on-going relationships so as to re-enter the job market.

The termination of relationships entails a risk though; it is assumed that employers and workers are paired through a successful outcome of an employment lottery. Thus, participating in the contract market does not guarantee that a relationship will be entered right away.[6] In this context, employment relationships are self-enforcing if workers and employers have no incentive to terminate them in the hope of entering more favourable ones. Since contracts offered on the job market are themselves affected by the possibility that agents might renege, the value of outside opportunities is determined endogenously. This feature of the model improves upon the equilibrium considered in Thomas and Worrall (1988). In their model, agents who renege once, do not have access to the market for employment relationships anymore; they are compelled to trade labour through anonymous spot transactions.[7] Contracts are thus enforced by permanent exclusion. Punishments are only temporary in the present paper. Upon reneging, agents must participate in the employment lottery but they are not permanently excluded from ever entering another employment relationship; bygones are bygones.

The behaviour of earnings implied by the model is nevertheless qualitatively similar to that described in Thomas and Worrall (1988): A worker's earnings grow at the average

[5] The approach of Gomme and Greenwood (1995) is quite different from that of the other papers. Gomme and Greenwood (1995) study labour contracts that support equilibria of a system of complete Arrow-Debreu markets. Their framework does not allow contracts to have real effects. Danthine and Donaldson (1992) and Boldrin and Horvath (1995), who rely on implicit contract theory to describe labour relations, allow contracts to have real effects similar to those described by Rosen (1985).

[6] This matching friction supports the existence of self-enforcing employment relationships which involve some degree of risk-sharing. Such relationships are not possible with a frictionless job market. As another match can be found right away in this case, opportunistic behaviour remains unpunished. Therefore, a spot market equilibrium obtains.

[7] Essentially, Thomas and Worrall (1988) consider a labour market composed of an anonymous Walrasian spot market and of a market for long-term employment contracts. The existence of this dual market is based on the assumption that two types of workers exist: relatively patient workers and relatively impatient workers (casual workers) who discount the future so much that they do not wish to participate in the market for long-term labour contracts.

growth rate of productivity as long as such increase does not induce he or his employer to renege. In this case, perfect sharing of all insurable risks is achieved. However, if increasing earnings at this rate would lead one of the parties to renege, then earnings are set at a level (higher or lower than that implied by average productivity growth) which ensures the continuation of the relationship. The frequency at which this latter situation occurs depends on how attractive employment relationships are compared to outside opportunities. This is, in turn, closely related to the degree of ease with which matches are formed. On the one hand, if the job market is relatively frictionless, punishments for reneging are short-lived as a new match can be found relatively easily. In this case, the model's equilibrium resembles that implied by a Walrasian labour market. On the other hand, if matching frictions are important, the equilibrium is similar to that of an economy with full commitment since agents have little incentives to renege in this case.

The two polar cases used to describe labour relations in the literature, that is, the contract economy with full commitment and the Walrasian auction market, are therefore nested within this model. Hence, the model provides a unified framework within which the implications of limited commitment can be studied. Three sets of results are presented. First, it is shown that the model performs very well in replicating labour market fluctuations. The behaviour of wages, hours worked, average productivity, and of the labour share predicted by the model are quite similar to that observed in the U.S. economy. In particular, real wages are quite persistent and mildly procyclical while hours worked are strongly procyclical. Second, the model's properties are compared to those of a model in which full commitment is assumed. These results suggest that a model with limited commitment offers a better performance, especially in its account of the comovements of hours worked and real wages. Finally, the paper evaluates whether neglecting the fact that earnings embody an insurance component introduces a significant bias in estimates of labour supply elasticities. The results suggest that this is the case.

The remainder of the paper is organized as follows. The model is described in Section 2. Section 3 describes the properties of equilibrium self-enforcing employment relationships in this context. Section 4 discusses a plausible parameterization of the model. Simulation results are presented in Section 5. Section 5 compares the properties of the model with those of key U.S macroeconomic time series. It also examines issues related to the estimation of labour supply elasticities using panel data. The last section offers concluding remarks.

2. The model.

Technology and preferences

Consider an economy populated with identical risk-neutral entrepreneurs and identical risk-averse workers. Entrepreneurs and workers are infinitely lived. Entrepreneur have access to perfect capital markets but workers do not; they cannot save or borrow. This setting is common in the implicit contract literature; Danthine and Donaldson (1992) and Boldrin and Horvath (1995) make similar assumptions.[8] Based on personal consumption and earnings data, Beaudry and Pages (1999) argue that this assumption is reasonable if one is only interested in movements along the business cycle. Each entrepreneur (or firm) has two constant returns to scale production technologies at his disposal; a market technology which requires the services of exactly one worker and a nonmarket technology which does not require any labour input. Each entrepreneur is also endowed with K units of a nontradable and indivisible factor of production. Both production technologies require k units of this input to operate.[9] It is assumed that $K = k$. Thus entrepreneurs must effectively decide which technology to operate; both technologies cannot be used at the same time.

The number of entrepreneurs is assumed to exceed the number of workers.[10] Since each entrepreneur can hire at most one worker, there are more potential jobs than there are workers. Thus, an entrepreneur's decision to operate the market technology is akin to that of a firm to create a job. The market production technology takes the standard form $A^t\tilde{\theta}_t h_t^{\alpha} k_t^{1-\alpha}$ in which $\tilde{\theta}_t$ is a stationary random variables, h_t denotes hours worked, and A^t ($A \geq 1$) is a deterministic growth component. The nonmarket technology is $r(\tilde{\theta}_t)A^t k_t$ in which $r(\tilde{\theta}_t)$ is a non-negative function of $\tilde{\theta}_t$. In both technologies, k_t is either equal to k or 0.[11] The random variable $\tilde{\theta}_t$ follows a first-order Markov process with set of possible states Θ and transition matrix Π. The set of states $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$ is such that $\theta_{i+1} > \theta_i > 0$ and the

[8] See also Thomas and Worrall (1988), Beaudry and DiNardo (1991, 1995), and MacLeod and Malcomson (1998).

[9] One may think of this factor of production as entrepreneurial skills. The introduction of this factor of production ensures that overall returns to scale are constant in both technologies but allows for decreasing marginal product of labour in the labour-intensive technology.

[10] Alternatively, one could assume a small number of entrepreneurs who can employ several workers. This would lead to equivalent results if entrepreneurs do not consider employment contracts signed in the past when they offer new ones.

[11] For simplicity, the model does not allow for the accumulation of capital. Allowing for capital accumulation would significantly complicate the analysis. In this case, agents' decisions would not only depend on the capital stock accumulated "within" a particular relationship, but also on the aggregate capital stock (through their respective outside opportunities). As there is a fair amount of *ex post* heterogeneity in the model (see description of the labour market), finding the model's equilibrium would be quite difficult. Moreover, note that risk-sharing itself may affect investment decisions. Sigouin (1999) shows that risk-sharing may lead firms to overinvest.

matrix Π has typical elements $\pi_{si} = \Pr(\tilde{\theta}_{t+1} = \theta_i \mid \tilde{\theta}_t = \theta_s)$. Realizations of $\tilde{\theta}_t$ are common to all entrepreneurs. They are observed by all agents, at the beginning of each period, before any decisions are made.

Each worker's total time endowment is normalized to one. A worker has preferences over stochastic sequences of consumption and leisure $\{C_\tau, 1-h_\tau\}_{\tau=t}^{\infty}$ described by

$$\mathrm{E}_t \sum_{\tau=t}^{\infty} \beta_w^{\tau-t} [\ln(C_\tau) + B(1-\eta)^{-1}(1-h_\tau)^{1-\eta}] \tag{1}$$

in which E_t denotes the expectation operator conditional on information available in period t and $\beta_w \in (0,1)$ is the worker's discount factor.[12] Each entrepreneur values stochastic sequences of cash flows arising from the operation of the market technology according to

$$\mathrm{E}_t \sum_{\tau=t}^{\infty} \beta_e^{\tau-t} [A^\tau \tilde{\theta}_\tau h_\tau^\alpha k^{1-\alpha} - W_\tau h_\tau] \tag{2}$$

in which W_τ is the hourly wage rate paid to a worker in period τ and $\beta_e \in (0,1)$ is the entrepreneur's discount factor. It is assumed that $\beta_e A < 1$. Note that $C_\tau = W_\tau h_\tau$ since workers do not have access to capital markets.

The preferences described by (1) imply that hours are constant along a balanced growth path (See King, Plosser, and Rebelo 1988). This allows one to rewrite (1) and (2) in terms of rescaled quantities and effectively transform this growth economy into a no growth stationary economy. That is, let $c_t = C_t/A^t$ and $w_t = W_t/A^t$; then,

$$U_t = \mathrm{E}_t \sum_{\tau=t}^{\infty} \beta_w^{\tau-t} [\ln(c_\tau) + B(1-\eta)^{-1}(1-h_\tau)^{1-\eta}] \tag{1$'$}$$

and

$$f_t = \mathrm{E}_t \sum_{\tau=t}^{\infty} (\beta_e A)^{\tau-t} [\tilde{\theta}_\tau h_\tau^\alpha k^{1-\alpha} - w_\tau h_\tau] \tag{2$'$}$$

are equivalent to (1) and (2) above.[13] It is more convenient to work with these expressions and recover W_t from w_t thereafter than to work directly with (1) and (2). Note that based on ($2'$), an entrepreneur operates the market technology in period t as long as f_t exceeds the present value of operating the nonmarket technology $V(\tilde{\theta}_t) = \mathrm{E}_t \sum_{\tau=t}^{\infty} (\beta_e A)^{\tau-t} r(\tilde{\theta}_\tau) k$.

The labour market

[12] This specification of workers' preferences implicitly assumes that the intertemporal elasticity of substitution is one. This choice is consistent with the findings of Beaudry and Van Wincoop (1996). It is a common choice in the RBC literature (e.g. see Andolfatto 1996)

[13] Note that a superfluous constant term $\beta_w \ln(A)/(1-\beta_w)^2$ has been dropped from ($1'$).

The fact that the market technology is affected by stochastic shocks implies that labour income may fluctuate over time. Risk-averse workers would like to insure themselves against potential income fluctuations but are unable to do so because they do not have access to capital markets. Being risk-neutral, entrepreneurs are ready to include an insurance component to wage payments if in return, workers agree to lower average earnings. Hence, gains from trade may be achieved through risk-sharing employment relationships. However, it is assumed that workers and entrepreneurs can terminate employment relationships at will. That is, quitting or firing is costless and neither actions can be prevented through the intervention of a third party. In this context, employment relationships must be disciplined by outside opportunities. Employment relationships are feasible only to the extent that workers have no incentives to quit and entrepreneurs have no incentives to lay off their employees. That is, employment relationships must be self-enforcing.

In any period, workers are either employed or unemployed. Entrepreneurs with a job vacancy can only hire workers from the pool of unemployed workers. Jobs are allocated through an employment lottery. An unemployed worker is paired with an entrepreneur with probability $\mu \in (0, 1]$ and remains unemployed with probability $1–\mu$. Workers who remain unemployed get unemployment benefits $\omega(\tilde{\theta})$. These benefits are financed through a lump-sum tax on firms' profits (i.e. on entrepreneurs operating the market technology). Entrepreneurs who are matched with a worker operate the nonmarket technology. Entrepreneurs and workers who are paired with each other enter employment relationships of indefinite length. These relationships are subject to random permanent separation shocks. That is, it is assumed that an on-going employment relationship ends in any period with probability $1-\delta$ for reasons outside of the agents' control. In such occurrences, the agents involved re-enter the job market.[14]

As there are more entrepreneurs (jobs) than workers, there is perfect competition among entrepreneurs in the job market. Therefore, in equilibrium, all entrepreneurs offer the same employment contract in a given period. Employment contracts specify sequences of wages and hours worked for as long as separation does not occur. These are determined after the current realization of $\tilde{\theta}$ has been observed and are potentially state-contingent since all agents share the same information. Thus, workers and entrepreneurs paired in different

[14] This setup of the labour market may appear similar to that of MacLeod and Malcomson (1998) who study self-enforcing employment contracts in a shirking model. It is quite different. The employment contracts developed therein do not involve any sharing of risks and the existence of these contracts rests on there being a short side of the labour market. Without risk-sharing, equilibrium wages are indeterminate. MacLeod and Malcomson (1998) need to revert to a social norm in order to determine equilibrium wages. This is not necessary with risk-sharing; equilibrium wages reflect the fact that workers attempt to smooth earnings. Moreover, there are no unfilled job vacancies in their setup. Therefore, the equilibrium considered has agents on the short side of the labour market receiving their reservation utility levels at all times. This need not be the case when there are unfilled vacancies.

periods possibly enter employment relationships which specify different sequences of wages and hours. This creates incentives to terminate relationships. Workers and entrepreneurs that are part of on-going relationships may wish to participate in the employment lottery if the relationship currently offered on the market is more attractive than remaining with their current relationships. Since quitting or firing is costless, nothing prevents agents from behaving opportunistically. Therefore, a relationship is self-enforcing if it ensures that neither of its parties are willing to run the risk associated with re-entering the job market.[15]

Employment relationships under limited commitment.

Let $u_s^U = \ln(\omega(\theta_s)) + B(1-\eta)^{-1}$, for $s = 1, 2, \ldots, n$, and define u^U as a n-dimensional vector with typical element u_s^U. Each element u_s^U of u^U corresponds to the momentary utility of an unemployed worker who does not get a job offer when the current realization of $\tilde{\theta}_t$ is $\theta_s \in \Theta$.[16] Define U_s^H as the present value of expected utility obtained by an unemployed worker (at the time of hiring) if he accepts a job offer when $\tilde{\theta}_t = \theta_s$ and let U^H be the n-dimensional vector formed from all the U_s^H. Competition among entrepreneurs determines the value taken by each U_s^H; this value is such that the present value attached by an entrepreneur to an employment relationship is no less than that of operating the nonmarket technology while waiting for another match.

A worker who enters the job market in any period is hired with probability μ and remains unemployed with probability $1-\mu$; in which case, he re-enters the job market next period and so on, until he is paired with an entrepreneur. It is easy to show that the present value of expected utility U_s^U obtained by an unemployed worker, as he enters the job market in state θ_s, is the s^{th} element of the n-dimensional vector

$$U^U = [I - \beta_w(1-\mu)\Pi]^{-1}[(1-\mu)u^U + \mu\, U^H] \tag{3}$$

in which I is an identity matrix of dimension n. Each element U_s^U of U^U is the worker's reservation utility level in state θ_s. Employment relationships cannot offer less than what U^U specifies because, in this case, workers would re-enter the job market. Similarly, workers cannot obtain more than what U^H specifies because entrepreneurs would then re-enter the job market.

Let U_{is}^C be the continuation utility promised to a worker by an on-going employment relationship in state θ_s, if no separation shock occurs in the following period, and if the state of nature is θ_i at that time. Then, the discounted expected utility of an employed worker in

[15] The only cost associated with the termination of a relationship arise from the fact that entering the employment lottery does not guarantee a match if $\mu < 1$.

[16] This implicitly assumes that workers do not spend any time searching for new jobs; they enjoy the maximal amount of leisure while unemployed.

state θ_s may be written as

$$U_s = \ln(w_s h_s) + B(1-\eta)^{-1}(1-h_s)^{1-\eta} + \beta_w \sum_{i=1}^{n} [\delta U_{is}^C + (1-\delta) U_i^U] \pi_{si} \tag{4}$$

in which w_s and h_s respectively denote the hourly wage and total hours worked in state θ_s. A relationship is self-enforcing in state θ_s, from the workers's standpoint, if $U_s \geq U_s^U$ and if for all possible continuations of the relationship, $U_{is}^C \geq U_i^U$. Similarly, the relationship is self-enforcing from the entrepreneur's standpoint if these utility levels ensure that he is no worse off than entering the employment lottery and operating the nonmarket technology in the meantime (both currently and in every possible continuations of the relationship). When this set of constraints is satisfied, no agent has an incentive to terminate the employment relationship.

The equilibrium values of h_s, w_s, and all U_{is}^C for each θ_s are determined by finding the set of Pareto efficient self-enforcing trades between an entrepreneur and a worker who are paired with each other. This is done by finding the Pareto frontier. A point on the Pareto frontier can be found by maximizing the present value of expected cash flows that an entrepreneur receives for a given reservation level of the worker's present value of expected utility. The entire Pareto frontier can be traced by varying the level of the worker's reservation utility level. As in Thomas and Worrall (1988), it is possible to define the Pareto frontier recursively by treating the worker's expected utility level as a state variable. Finding the Pareto frontier in this case amounts to solving a dynamic programming problem.

Define $F_s(U)$ as the Pareto frontier when the current state of nature is θ_s and the worker's reservation utility is U and consider the following dynamic programming problem

$$\begin{aligned} f_s(\tilde{U}) = \max_{h,w,\{\tilde{U}_i^C\}_i} & \left\{ \theta_s h^\alpha k^{1-\alpha} - wh - r(\theta_s)k + \beta_e A \delta \sum_{i=1}^{n} f_i(\tilde{U}_i^C) \pi_{si} \right\} \\ \text{s.t. } & \ln(wh) + B(1-\eta)^{-1}(1-h)^{1-\eta} + \beta_w \delta \sum_{i=1}^{n} \tilde{U}_i^C \pi_{si} \geq \tilde{U} \\ & \tilde{U}_i^C \geq \tilde{U}_i^U, \quad i = 1, 2, \ldots, n, \\ & f_i(\tilde{U}_i^C) \geq 0, \quad i = 1, 2, \ldots, n, \end{aligned} \tag{5}$$

in which

$$\tilde{U}^U = [I - \beta_w(1-\delta)\Pi[I-\beta_w\delta\Pi]^{-1}][I-\beta_w(1-\mu)\Pi - \mu\beta_w(1-\delta)\Pi[I-\beta_w\delta\Pi]^{-1}]^{-1}[(1-\mu)u^U + \mu\,\tilde{U}^H]$$

where $\tilde{U}^H = (\tilde{U}_1^H, \tilde{U}_2^H, \ldots, \tilde{U}_n^H)$ is such that $f_i(\tilde{U}_i^H) = 0$ for $i = 1, 2, \ldots, n$. Let π_s be the s^{th} row of the transition matrix Π. It is shown in Appendix A that the Pareto frontier can be obtained from $F_s(U) = f_s(U - \beta_w(1-\delta)\pi_s[I - \beta_w\delta\Pi]^{-1}U^U) + V(\theta_s)$. Recall that $V(\theta_s)$ is the entrepreneur's valuation of operating the nonmarket technology in state s. Thus, $f_s(\tilde{U})$ corresponds to the entrepreneur's valuation of the employment relationship net of operating the nonmarket technology. The value of $\tilde{U}$ corresponds to the worker's expected utility level obtained through the contract net of the expected utility associated with entering the job market (for exogenous reasons) at a later date.

The optimal employment relationship can be found by solving (5) for a given $\tilde{U}$. The first constraint in (5) ensures that the worker gets no less than the expected utility level he is promised by the relationship. The last two constraints in (5) are the agents' respective self-enforcing constraints; they ensure that both agents obtain from the relationship no less than what their respective outside opportunities offer. These two constraints would be omitted if it was somehow possible for both agents to commit not to terminate the relationship under any circumstances. The ensuing full-commitment Pareto frontier $F_s^*(U)$ describing efficient trades between a worker and an entrepreneur can be found, in a similar manner as $F_s(U)$, by solving

$$\begin{aligned} f_s^*(\tilde{U}) = \max_{h,w,\{\tilde{U}_i^C\}_i} & \Big\{\theta_s h^\alpha k^{1-\alpha} - wh - r(\theta_s)k + \beta_e A\delta \sum_{i=1}^n f_i^*(\tilde{U}_i^C)\pi_{si}\Big\} \\ \text{s.t. } & \ln(wh) + B(1-\eta)^{-1}(1-h)^{1-\eta} + \beta_w \delta \sum_{i=1}^n \tilde{U}_i^C \pi_{si} \geq \tilde{U} \\ & 1 \geq h \geq 0. \end{aligned} \tag{6}$$

As for $F_s(U)$, it is the case that $F_s^*(U) = f_s^*(U - \beta_w(1-\delta)\pi_s[I - \beta_w\delta\Pi]^{-1}U^U) + V(\theta_s)$. The dynamic programming problem in (6) can be solved using standard techniques; the one in (5) cannot.

The dynamic programming problem in (5) is not standard: The value function $f_s(\tilde{U})$ is only defined for values of $\tilde{U}$ that satisfy $\tilde{U} \geq \tilde{U}_s^U$ and $f_s(\tilde{U}) \geq 0$. Since $f_s(\tilde{U})$ is decreasing in $\tilde{U}$, this last requirement corresponds to $\tilde{U}_s^H \geq \tilde{U}$. Thus, the relevant domain of the value function in state θ_s is $[\tilde{U}_s^U, \tilde{U}_s^H]$; $f_s(\tilde{U})$ does not exist for values of $\tilde{U}$ outside this interval because in this case, either the worker or the entrepreneur finds it desirable to renege. Moreover, the vector of hiring utility levels $\tilde{U}^H$ is determined using the optimum value function itself. Since $f_s(\tilde{U})$ is unknown *a priori*, both vectors $\tilde{U}^H$ and $\tilde{U}^U$ are also unknown *a priori*. This is problematic because the relevant domain of the value function is itself determined by these vectors. Hence, in a value function iteration algorithm, each new update of the value function has its own domain. Each new domain does not necessarily coincide with that associated with previous updates of the value function. This makes the task of finding a fixed point in the space of value functions quite difficult, but not impossible. Thomas and Worrall (1994) show, for a similar dynamic programming problem, that a value function like $f_s(\,\cdot\,)$ in (5) may be found as the (pointwise) limit of an iterative scheme initialized with $f_s^*(\cdot)$. They also show that the resulting function is strictly decreasing and strictly concave on the interior of its domain, that is, on $(\tilde{U}_s^U, \tilde{U}_s^H)$ in the present case.[17]

3. Equilibrium employment agreement.

Consider the dynamic programming problem (5). Let the multipliers associated with each constraint be given by λ, $\{\beta_e \delta \phi_i \pi_{si}\}_{i=1}^n$, and $\{\beta_e \delta \psi_i \pi_{si}\}_{i=1}^n$ respectively. Given the optimal value function $f_s(\cdot)$, the solution of the static maximization problem in (5) is characterized by the following first-order conditions

$$-h + \lambda w^{-1} = 0, \tag{7}$$

$$\alpha \theta_s h^{\alpha-1} k^{1-\alpha} - w + \lambda[h^{-1} - B(1-h)^{-\eta}] = 0, \tag{8}$$

$$(1+\psi_i) f_i'(\tilde{U}_i^C) + \phi_i + (\beta_w/\beta_e A)\lambda = 0, \qquad i = 1, 2, \ldots, n, \tag{9}$$

together with the relevant Kuhn-Tucker conditions corresponding to each inequality constraint in (5). In addition, the envelope condition is

$$f_s'(\tilde{U}) = -\lambda. \tag{10}$$

Let $\underline{\mathrm{w}}(\theta_s) = -f_s'(\tilde{U}_s^U)$ and $\bar{\mathrm{w}}(\theta_s) = -f_s'(\tilde{U}_s^H)$. Since $\tilde{U}$ necessarily belongs to $[\tilde{U}_s^U, \tilde{U}_s^H]$, equation (7) and (10), together with the fact that $f_s(\cdot)$ is concave, imply that $wh \in [\underline{\mathrm{w}}(\theta_s), \bar{\mathrm{w}}(\theta_s)]$. The interval $[\underline{\mathrm{w}}(\theta_s), \bar{\mathrm{w}}(\theta_s)]$ forms the range of feasible (self-enforcing) earnings of the worker when $\tilde{\theta}_t = \theta_s$. Any value of the worker's total income below $\underline{\mathrm{w}}(\theta_s)$ would prompt him to renege. Similarly, any value of wh above $\bar{\mathrm{w}}(\theta_s)$ would lead the entrepreneur to renege.

Prior to describing the behaviour of wages and hours implied by (7)-(10), it is useful to describe their behaviour under full commitment. This is done in Proposition 1 below.

Proposition 1. Let $\nu = \beta_w/\beta_e$. Consider an employment relationship, beginning in period τ and ending in period $T+1$ (for exogenous reasons), described by a sequence of wage payments and hours worked $\{W_t, h_t\}_{t=\tau}^T$ for a given history of shocks $\{\tilde{\theta}_t\}_{t=\tau}^T$. Moreover, for all $s = 1, 2, \ldots, n$, let $w^*(\theta_s) = -f^{*\prime}(U_s^*)$ in which U_s^* solves $f_s^*(U_s^*) = 0$. Then, under full commitment, the sequence $\{W_t, h_t\}_{t=\tau}^T$ is characterized by:

$$W_\tau h_\tau = A^\tau w^*(\tilde{\theta}_\tau), \tag{11}$$

$$W_t h_t = \nu W_{t-1} h_{t-1}, \qquad T \geq t \geq \tau+1 \tag{12}$$

and for $h_t \in [0, 1]$,

$$W_t h_t B(1-h_t)^{-\eta} = \alpha A^t \tilde{\theta}_t h_t^{\alpha-1} k^{1-\alpha}. \tag{13}$$

[17] See Appendix A for more details

Proof. Follows directly from the first-order conditions and the envelope condition associated with (6). These are the same as (7)-(10) (with f replaced by f^*) with $\psi_i = \phi_i = 0$ for all i. Equation (12) is obtained by combining (7), (10), and (9) (with $\psi_i = \phi_i = 0$ for all i). Equation (13) follows directly from (7) and (8). W_t is obtained from w by multiplying by A^t. ‖

Equation (11) in Proposition 1 determines the hiring wage implied by competition among entrepreneurs. Equation (12) states that, under full commitment, workers' earnings grow at rate ν regardless of the temporary fluctuations in labour productivity. Finally, equation (13) is the familiar efficiency condition stating that the marginal rate of substitution between leisure and consumption is equated with the marginal product of labour. This equation determines hours worked in any period. It is clear from (12) and (13) that unless $\nu = A$, hours worked decline or increase steadily over time. This is inconsistent with the behaviour of hours per worker in the U.S. economy.[18] The only case of interest under full commitment is thus when $\nu = A$ but this entails a restriction on workers' and entrepreneurs' relative degrees of impatience. When $\nu = A$, earnings grow, on average, at the same rate as productivity. Moreover, exogenous variations in productivity lead to relatively elastic responses of hours worked. This occurs, in this case, because optimal risk-sharing smooths labour income (consumption). Therefore, $W_t h_t$ is essentially predetermined in (13); this effectively eliminates the income effects of a change in productivity and promotes an elastic response of labour supply (see Rosen 1985 for a detailed exposition).

To some extent, the behaviour of earnings and hours worked described by Proposition 1 may be compared to that in Boldrin and Horvath (1995) and Gomme and Greenwood (1995). Gomme and Greenwood (1995) examine labour contracts which influence the behaviour of factor payments but do not affect real allocations. In the present context, this would correspond to a situation where hours worked were constrained to be equal to that observed under a Walrasian equilibrium (i.e. equal to the value h_* that solves $h_* B(1-h_*)^{-\eta} = 1$). An equation similar to (12) would describe the behaviour of earnings (or hourly wage since $h_t = h_{t-1} = h_*$) except that ν would be replaced by a time-varying component ν_t (Gomme and Greenwood 1995 assume time-varying endogenous discount factors).[19] Boldrin and Horvath (1995) examine one-period labour contracts which are negotiated one period in advance. Their model essentially leads to similar results to those embodied in (12); earnings are not contingent and there is thereby no income effect involved in the determination of hours worked (see equation (13)).

[18] This fact usually motivates the use of preferences that guarantee constant hours along a balanced growth path in the RBC framework.

[19] This seems confirmed by comparing Table 2 and Table 5 in Gomme and Greenwood (1995). Once discount factors are assumed constant, the variability of real wages is reduced substantially and their first-order autocorrelation increases.

The behaviour of earnings and hours worked differs under partial commitment since self-enforcing constraints have to be taken into account. Proposition 2 below summarizes the implications of partial commitment for the employment relationship.

Proposition 2. Let $\nu = \beta_w/\beta_e$ and let h_* be such that $h_* B(1-h_*)^{-\eta} = 1$. Consider an employment relationship, beginning in period τ and ending in period $T+1$ (for exogenous reasons), described by a sequence of wage payments and hours worked $\{W_t, h_t\}_{t=\tau}^{T}$ for a given history of shocks $\{\tilde{\theta}_t\}_{t=\tau}^{T}$. Then, under partial commitment,

i) if $\mu < 1$, the sequence $\{W_t, h_t\}_{t=\tau}^{T}$ is characterized by:

$$W_\tau h_\tau = A^\tau \bar{\mathrm{w}}(\tilde{\theta}_\tau), \tag{14}$$

$$W_t h_t = \begin{cases} A^t \bar{\mathrm{w}}(\tilde{\theta}_t), & \text{if } \nu W_{t-1}h_{t-1} > A^t \bar{\mathrm{w}}(\tilde{\theta}_t), \\ \nu W_{t-1}h_{t-1}, & \text{if } A^t \bar{\mathrm{w}}(\tilde{\theta}_t) \geq \nu W_{t-1}h_{t-1} \geq A^t \underline{\mathrm{w}}(\tilde{\theta}_t), \\ A^t \underline{\mathrm{w}}(\tilde{\theta}_t), & \text{if } A^t \underline{\mathrm{w}}(\tilde{\theta}_t) > \nu W_{t-1}h_{t-1}, \end{cases} \qquad T \geq t \geq \tau+1 \tag{15}$$

and

$$W_t h_t B(1-h_t)^{-\eta} = \alpha A^t \tilde{\theta}_t h_t^{\alpha-1} k^{1-\alpha}; \tag{16}$$

ii) if $\mu = 1$ and $r(\tilde{\theta}_t) = (1-\alpha)\tilde{\theta}_t h_*^{\alpha} k^{-\alpha}$, the sequence $\{W_t, h_t\}_{t=\tau}^{T}$ is characterized by:

$$h_t = h_*, \tag{17}$$

$$W_t = \alpha A^t \tilde{\theta}_t h_*^{\alpha-1} k^{1-\alpha}. \tag{18}$$

Proof. Part i): Equation (15) results from a straightforward extension of Proposition 2 in Thomas and Worrall (1988). Equation (16) obtains from (7) and (8). Part ii): If $\mu = 1$, then it is readily seen from the definition of $\tilde{U}^U$ that $\tilde{U}^U = \tilde{U}^H$. Thus, only $\tilde{U}_i^C = \tilde{U}_i^H$ for all i satisfies both self-enforcing constraints in (5). Since the hiring utility level is chosen from $\tilde{U}^H$, the contract delivers only values from $\tilde{U}^H$ over time. Hence, the entrepreneur's net valuation of the contract must be null at all times and no risk-sharing occurs. Equations (17) and (18) follow and $r(\tilde{\theta}_t) = (1-\alpha)\tilde{\theta}_t h_*^{\alpha} k^{-\alpha}$ ensures that there is indeed a vector $\tilde{U}^H$ such that $f_i(\tilde{U}_i^H) = 0$ for all i. ||

The first part of Proposition 2 describes the movements of wage and hours implied by a self-enforcing relationship. Equation (14) is the equivalent of equation (11); it determines initial wages. Equation (15) depicts a now familiar result in self-enforcing implicit contract theory (e.g. see Thomas and Worrall 1988); full-commitment or perfect risk-sharing payments are implemented in any period unless this would prompt one of the agents to renege. The interval $[A^t \underline{\mathrm{w}}(\tilde{\theta}_t), A^t \bar{\mathrm{w}}(\tilde{\theta}_t)]$ forms the range of labour income for which both agents have no incentives to renege in period t ; $\nu W_{t-1}h_{t-1}$ is the level of earnings that would prevail at that time if agents were able to commit fully (see Proposition 1). If full-commitment earnings lie within this interval, they are implemented. Otherwise, either the lower or the upper bound of this interval is implemented. That is, workers' earnings deviate from their full-commitment value by the least amount possible that keeps both agents from reneging. This is essentially implied by the fact that workers are more risk-averse than entrepreneurs.

The worker's income under the relationship is fully determined by (15) and the initial value (14). Note that the whole sequence of labour income depends on the value of $\tilde{\theta}$ at the time of hiring; workers hired in different time periods may have different earning profiles even if realizations of $\tilde{\theta}$ are common to all.[20] Equation (16) determines hours worked under partial commitment. Note that it is identical to equation (13). However, in this case, the value of ν need not be equal to A in order to have an interior solution for hours worked. The bounds in (15) ensures that hours do not converge to either 0 or 1 over time since they keep earnings from systematically increasing (decreasing) at a faster (slower) rate than productivity. In addition, while as for equation (11), equation (14) also implies that earnings are essentially predetermined, the fact that labour income is bounded in any period implies that income effects are not necessarily eliminated from (16).

For instance, suppose for simplicity that $\nu = A = 1$ and let ϵ be the intertemporal elasticity of labour supply. Consider the effect on hours worked of an exogenous increase in productivity from period $t-1$ to period t by $z\%$. If $W_{t-1}h_{t-1}$ belongs to the interval $[\underline{\mathrm{w}}(\tilde{\theta}_t), \bar{\mathrm{w}}(\tilde{\theta}_t)]$, then this increase in productivity does not involve an income effect since $W_t h_t = W_{t-1}h_{t-1}$ in this case. This leads to an increase in hours worked of approximately $z\epsilon/(1+\epsilon(1-\alpha))\%$ (see equation (16)) and a decrease in hourly wage of the same magnitude. However, if $W_{t-1}h_{t-1} < \underline{\mathrm{w}}(\tilde{\theta}_t)$, labour income increases with respect to its previous value and the productivity increment involves income effects. In this case, hours worked increase by less than $z\epsilon/(1+\epsilon(1-\alpha))\%$ and the response of the hourly wage is ambiguous. The first case examined is more likely to occur when changes in productivity are modest whereas the latter case is more likely to be associated with large fluctuations in productivity. Hence, if partial commitment characterizes employment relationships, small increases in productivity should be associated with proportionally large increases in hours worked and decreases in hourly wages. Relatively large increases in productivity should lead to relatively modest increases in hours worked and, most likely, increases in hourly wages.

The second part of Proposition 2 describes conditions under which the only feasible self-enforcing employment relationship entails spot transactions uniquely. Equations (17) and (18) correspond to the equilibrium conditions that would obtain in an otherwise similar economy where the possibility of multi-period contracting is not considered; labour would

[20] This is also true in the case with full commitment. This observation is the basis for Beaudry and DiNardo's (1991) empirical investigation aimed at assessing the importance of enforcement considerations for labour contracts.

be paid its marginal product at all times and hours worked would be constant. This part of Proposition 1 underlies what makes risk-sharing employment relationships possible in this model; frictional unemployment. If $\mu = 1$, terminating a relationship entails no risks; re-entering the job market yields a new match right away. In this context, a relationship that would offer less than the current hiring utility level offered on the market, would immediately be terminated by workers. Similarly, entrepreneurs cannot credibly promise more than the hiring utility level offered on the market. When $\mu < 1$, however, breaking up on-going employment relationships is risky. Enduring them may thus be worthwhile. This is simply a restatement of Rosen's (1985) observation that "Contract markets are supported by frictions and specificity of employment relationships that tend to insulate contracting parties from short-run external shocks (...)", that is, from short-run incentives to renege.

The model is examined further by obtaining a numerical solution of (5).[21] In order to do so, a value must be assigned to each parameter of the model. This is the purpose of the next section.

4. Parameterization.

In all, the value of nine parameters has to be determined in addition to the set of possible states of nature Θ, the Markov transition matrix Π, and the functions $\omega(\tilde{\theta})$ and $r(\tilde{\theta})$. Four parameters pertain to preferences (β_e, β_w, η, and B), two parameters are key determinants of job creation and destruction (μ, δ) in the artificial economy, and three parameters pertain to the production technologies (α, A, and k). The model is considered at a quarterly frequency. Entrepreneurs' discount factor β_e is set to 0.99, which is consistent with a real interest rate of one percent per quarter. Workers' discount factor β_w is set to 0.993 and $A = \beta_w/\beta_e$ is used. This value of A implies that $\nu = A$ in equations (12) and (15). Therefore, full-commitment earnings grow at the same rate as productivity. It also implies that productivity grows at an average annual rate of 1.2%, a value consistent with that observed in the U.S. during the post-Korean war period. The labour share of income, α, is set at 0.64, its average value during the same period. These are all standard parameters' values in the real business cycle literature. The value of k is normalized to 1.

Bansak and Raphael (1998) report that employment relationships in the U.S. end within their first year of existence with an average probability of 0.206. On a quarterly basis, this corresponds to a separation probability of 0.056.[22] Accordingly, δ is set equal to 0.944. Jones and Riddell (1998) report a monthly transition probability from unemployment to employment of 0.261 in the U.S. from 1979 to 1994. On a quarterly basis, this corresponds

[21] A variant of the value function iteration algorithm is used. The value function at each iteration is approximated by a shape-preserving piecewise cubic Hermite interpolant. Details of the procedure are available in Sigouin (1999).

[22] That is, $\delta = 1 - (1 - 0.206)^{0.25}$.

to a transition probability of 0.60; thus, μ is set to 0.60.[23] Together, the values of μ and δ yield job destruction and job creation rates similar to those used in Cole and Rogerson (1999). The implied average unemployment rate is 3.6%. This value is consistent with that of estimates of the frictional unemployment rate in U.S. manufacturing (see Warren 1991).

The value of B is set at $2^{\eta}3^{1-\eta}$. This implies that workers spend, on average, a third of their nonsleeping hours working and that $h_* = 1/3$. Choosing an appropriate value for the intertemporal labour supply elasticity is controversial. For example, Greenwood, Hercowitz, and Huffman (1988) argue that 1.7 is a reasonable value. However, as Pencavel's (1986) work indicates, microeconomic evidence points to much smaller values. Recently, for instance, Kimmel and Kniesner (1998) report an elasticity of labour supply of 0.39 for men and of 0.66 for women. The average (weighted by the population composition) is 0.51. However, these estimates are based on the assumption that wages are determined in spot markets. Beaudry and DiNardo (1995) estimate a labour supply equation derived from a model allowing for long-term employment relationships. Their results suggest a labour supply elasticity around 0.8 for men (in the present model's context).[24] In light of Kimmel and Kniesner's (1998) results concerning the relative labour supply elasticities of men and women, an average elasticity of 1.0 is assumed.[25] This requires that η be set to 2. Andolfatto (1996) uses a similar specification of workers' preferences.

The set of states of nature Θ and the transition matrix Π were chosen so as to approximate

$$\ln(\tilde{\theta}_t) = 0.95 \ln(\tilde{\theta}_{t-1}) + e_t, \tag{19}$$

for $e_t \sim \mathrm{N}(0, 0.00763^2)$. This specification of productivity shocks is in line with standard calibration of real business cycles models. The choice of appropriate values for Θ and Π was done following the method described in Deaton (1991). A total of nine states of nature is used (i.e. $n = 9$). Finally, the functional forms $\omega(\tilde{\theta}) = \omega$, for all $\tilde{\theta}$, and $r(\tilde{\theta}) = (1-\alpha)\tilde{\theta}h_*^{\alpha}k^{-\alpha}$ are used for $\omega(\,\cdot\,)$ and $r(\,\cdot\,)$. This choice of $r(\,\cdot\,)$ ensures that the spot market economy is nested within the economy with implicit contracts (if μ is set to unity). The value of ω is chosen so that unemployment benefits are, on average, 44% of the average quarterly earnings. This value is consistent with the U.S. average wage replacement rate.[26]

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23 That is, $\mu = 1 - (1 - 0.261)^3$.

24 Based on their equation (2), the labour supply elasticity can be computed as $-\Omega_1/(1 + (2-\alpha)\Omega_1)$ in this model. The pooled estimate of Ω_1 reported in table VI is used to obtain an elasticity of 0.8. Results in this table correspond to a labour market characterized by commitment problems.

25 The average elasticity can be computed as $(0.51/0.39) \times 0.8 \simeq 1.0$.

26 The average replacement rate is calculated as the ratio of average weekly unemployment benefits to average weekly wage of production and nonsupervisory workers. See Economic Report of the President, 1999, tables B-45 and B-47. This ratio has increase steadily over time. Note that the actual replacement ratio may be higher because of progressive income taxation.

5. Simulation results.

Macroeconomic comparison.

The set of parameter values described in the previous section is used to simulate 100 samples of 180 periods. Each simulation entails 2000 workers. Various moments are computed. The third column of Table 1 reports their average value over the 100 samples. The fourth and the fifth columns perform the same exercise for the case where commitment issues are disregarded (i.e. when the self-enforcing constraints in (5) are omitted) and the case where $\mu = 1$ (i.e. for the spot market economy) respectively.[27] Note that the moments presented in this table are calculated from the deviations from trend of the log of each variable indicated. In line with the real business cycle literature, deviations from trend are computed by applying the Hodrick-Prescott filter. The second column of table 1 reports corresponding moments for quarterly U.S. nonagricultural business sector data from 1954 to 1999.[28] The data was transformed in the same fashion as those of the artificial economies. When applicable, variables are expressed in terms of units per worker.[29] Panel A reports standard deviations relative to that of output per worker. Panel B reports correlations with output per worker. Finally, Panel C reports first-order autocorrelations.

Workers' preferences, and the fact that they do not have access to capital markets, imply that hours are constant in the spot market economy; the income effect exactly cancels the substitution effect in this case. As hours worked remain constant, earnings, real wages, and labour productivity are perfectly correlated with output, and the labour share of income is constant. Except for the behaviour of hours, this is qualitatively similar to what early RBC models predict. Income effects are completely eliminated when full commitment is assumed. Therefore, hours worked become quite sensitive to changes in productivity. This lead to an increase in the variability of output by more that 40%. However, there is an excessive degree of risk-sharing in this case. As earnings do not vary at all with changes in productivity, but hours do, productivity increases lead to reductions in the wage rate.[30] This is clearly inconsistent with U.S. data. There is no systematic relation between the two in U.S. data; the correlation between wages and hours is 0.03. Overall, the properties of the model with full commitment are qualitatively similar to those presented in Gomme and Greenwood (1995).

The model performs significantly better when the assumption of full commitment is dropped. While the model with limited commitment also predicts a negative correlation

[27] The case with $\mu = 0$ yields similar results as those obtained when full commitment is assumed.

[28] Appendix B provides a description of the data.

[29] This transformation minimizes the impact of movements in and out of employment on the series. As such movements are exogenous in the model, they are unlikely to mimic actual movements. The transformation makes artificial series more comparable to their observed counterpart.

[30] The standard deviation of earnings is different than zero because the wage of new hires vary with the prevailing productivity level (see Proposition 1).

between hours and wages, its magnitude is not inconsistent with that observed in the second half of the sample (-0.25). In addition, the wage-productivity and hours-productivity correlations are similar to those found in U.S. data. This is also true of the cyclical behaviour of wages and hours. The model with full commitment and the model with spot transactions lead to counterfactual predictions in this case. With limited commitment, wages are persistent and barely react to fluctuations in output, just as in U.S. data. Hours are procyclical but not persistent enough. This may be due to the fact that capital utilization does not vary in the model. Overall, the model's properties are in line with observed labour market fluctuations. It is important to note, though, that the presence of employment relationships primarily affect factor payments. Output dynamics are essentially unaffected. The behaviour of hours worked arising from these relationships only enhance its variability.

Table 2 examines the impact of varying the intertemporal elasticity of labour supply on the model's properties. The fourth column reproduces the results presented in Table 1 for the case of no commitment. The results in the fifth column can, to some extent, be compared with those presented in Gomme and Greenwood (1995) and Boldrin and Horvath (1995). Both papers use parameterizations of preferences which imply intertemporal labour supply elasticities exceeding 1.5. Increasing the labour supply elasticity improves the model's performance along some dimensions, notably, in its account of wages and hours comovements (see Panel D). Note, however, that an increase of the intertemporal elasticity of labour supply increases the correlation of real wages with output even if the substitution effect should, in principle, be stronger. This is due to the fact that increasing the intertemporal elasticity of labour supply also reduces the scope for risk-sharing, that is, the distance between $\underline{\mathrm{w}}(\tilde{\theta}_t)$ and $\bar{\mathrm{w}}(\tilde{\theta}_t)$, and thereby strengthen income effects. This is confirmed by an increase in the variability of earnings.

Estimation of the intertemporal elasticity of labour supply.

MaCurdy (1981) argues that intertemporal labour supply elasticities may be estimated, on panel data, using either one of the following equations:

$$\Delta\ln(h_{it}) = \alpha_0 + \alpha_1 \Delta\ln(W_{it}) + \varepsilon_{it}, \tag{20}$$

$$\Delta\ln(h_{it}) = \alpha_0 + \alpha_1 \Delta\ln(W_{it}h_{it}) + \varepsilon_{it}, \tag{21}$$

in which i and t index individuals and time respectively. These equations are derived from the life-cycle model of labour supply, a standard tool in the analysis of earnings and hours data. In equations (20) and (21), average hourly earnings W_{it} correspond to the marginal product of labour. The parameter of interest in equation (20) is α_1. It is $\alpha_1/(1-\alpha_1)$ in equation (21).[31] Both provide estimates of the intertemporal labour supply elasticity. That

[31] Equation (21) is obtained from equation (20) by adding $\alpha_1\Delta\ln(h_{it})$ on both sides of (20).

is, the elasticity of hours to wages, holding the marginal utility of consumption constant. Equation (21) may be preferred to equation (20) because it avoids the bias involved in computing average hourly earnings by dividing earnings by hours worked when hours worked are not reported accurately.[32]

In the absence of commitment, estimating equation (20), or equation (21), is unlikely to produce an accurate assessment of workers' willingness to substitute labour intertemporally. Indeed, denote worker i's marginal labour productivity $\alpha A^t \tilde{\theta}_t h_{it}^{\alpha-1} k^{1-\alpha}$ by W_{it}^*. Substituting equation (16) in Proposition 2 into equation (15) and using a log-linear approximation, yield the relationship

$$\Delta \ln(h_{it}) = \alpha_0 + \alpha_1 \Delta \ln(W_{it}^*) \tag{22}$$

only if the lagged value of earnings, $W_{it-1}h_{it-1}$, satisfies

$$A^t \bar{\mathrm{w}}(\tilde{\theta}_t)/\nu \geq W_{it-1}h_{it-1} \geq A^t \underline{\mathrm{w}}(\tilde{\theta}_t)/\nu. \tag{23}$$

Equation (22) and (23) highlight two potential problems with the estimation of equation (20). First, the existence of employment relationships imply that $W_{it} \neq W_{it}^*$. That is, hourly earnings do not measure labour productivity appropriately. Instrumental variables methods are unlikely to alleviate this problem. Second, equation (20) is mis-specified when lagged earnings do not satisfy (23). The first problem is related to risk-sharing uniquely whereas the latter problem arises from the absence of commitment.

In order to evaluate how severe these problems are for the estimation of labour supply elasticities, equations (20) and (21) are estimated on artificial data obtained from the model's simulations. In practice, it is common to use data from the Panel Study of Income Dynamics (PSID) to estimate labour supply elasticities. This dataset traces the yearly job history of individual workers for several consecutive years. Thus, the estimation of (20) and (21) is performed using variables equivalent to those available in the PSID. As most empirical studies based on the PSID use ten years of data, a subsample of forty quarters (10 years) is taken from each simulation of the model. Hours worked and earnings are summed over four consecutive quarters in order to obtain yearly quantities. Average hourly earnings are calculated by dividing annual earnings by annual hours. A point-in-time measure of hourly wage rates is also taken from the first quarter of each year. Tenure is computed as the number of years an individual worker has been part of a specific relationship. Experience is captured by a time trend. There is no need to account for other individual characteristics since, except for their job history, workers are all identical in the simulations. All these variables are consistent with those available from the PSID.

Table 2 reports average pooled IV estimates of α_1 and $\alpha_1/(1-\alpha_1)$ over 100 panel of 2000 workers. Each estimation if performed for different levels of intertemporal labour supply

[32] See Beaudry and DiNardo (1995) for a discussion of this bias.

elasticity. Panel A reports estimation results based on (20). Hourly earnings are instrumented with the point-in-time measure of wages. Altonji (1986) argues that this is a better instrument than the age and tenure data used in MaCurdy (1981). Panel B reports estimation results based on (21). Following, MaCurdy (1981) this time, hourly earnings are instrumented with a time trend, job tenure, and year dummy variables. Both panels present similar results, though those in Panel B appear slightly better. Average R-squared are consistent with those usually reported in the empirical literature. Overall, as standard deviations show, the range of estimates obtained varies substantially. On average, estimates are similar to those found in the literature (see Pencavel 1986). In all cases, however, they are well below actual values.

6. Conclusion.

This paper considers an economy in which agents cannot precommit to fulfill their engagements. Firms hire workers by offering them implicit employment contracts. The terms of these contracts are such that neither employers nor workers voluntarily choose to terminate their association in order to opt for other opportunities at their disposal. It is shown that, overall, labour market fluctuations implied by the model are consistent with those observed in the U.S. economy. In particular, the model does well in reproducing the comovements of hours, real wages, and labour productivity. Risk-sharing is responsible for an increase in the variability of output but does not have a significant impact on its dynamic behaviour. Hence, while self-enforcing employment relationships can help explain the behaviour of factor payments, their impact on the dynamics of the model are limited. If one is only interested in studying output dynamics, for example, then neglecting their existence may be harmless.

However, disregarding the existence of self-enforcing employment relationships when estimating labour supply elasticities may be harmful. It may lead one to underestimate workers' willingness to substitute leisure intertemporally. Hence, the statement that one needs to assume counterfactually high elasticities of labour supply to account for observed labour market fluctuations may be vacuous. If self-enforcing employment relationships indeed characterize labour relations, as the empirical work of Beaudry and DiNardo (1991, 1995) suggests, then new estimates of labour supply elasticities that are consistent with this fact, are required. Estimates which translate an increased willingness to substitute leisure intertemporally may help reconcile equilibrium business cycle models with observed fluctuations.

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Appendix A.

This appendix shows that the Pareto frontier can be found by solving problem (5). To this end, let $F : \mathcal{R}^n \rightarrow \mathcal{R}^n$ and denote by F_i the i^{th} element of F. Given the vector of hiring utility levels U^H, the Pareto frontier is the value function F with element F_s, $s = 1, 2, \ldots, n$, solving

$$\begin{aligned} F_s(U) = & \max_{h,w,U^C} \left\{ \theta_s h^\alpha k^{1-\alpha} - wh + \beta_e A \sum_{i=1}^{n} [\delta F_i(U_i^C) + (1-\delta) W_i] \pi_{si} \right\} \\ & \text{s.t. } \ln(wh) + B(1-\eta)^{-1}(1-h)^{1-\eta} + \beta_w \sum_{i=1}^{n} [\delta U_i^C + (1-\delta) U_i^U] \pi_{si} \geq U \\ & U^U = [I - \beta_w (1-\mu)\Pi]^{-1}[(1-\mu)u^U + \mu U^H], \\ & U_i^C \geq U_i^U, \quad i = 1, 2, \ldots, n, \\ & F_i(U_i^C) \geq W_i, \quad i = 1, 2, \ldots, n. \end{aligned} \tag{A1}$$

In the above dynamic programming problem, W_i is the i^{th} element of the vector W defined as

$$W = [I - \beta_e A(1-\mu)\Pi]^{-1}[(1-\mu)rk + \mu F(U^H)] \tag{A2}$$

in which $r = (r(\theta_1), r(\theta_2), \ldots, r(\theta_n))$. The vector W gives the entrepreneur's valuation of entering the job market and waiting for a new match.

i) By assumption (in the text), competition among entrepreneurs is such that $F(U^H) = W$. That is, the hiring utility level in any state of nature is such that the labour contract's net valuation is zero. Using this requirement in ($A2$) and solving for W (assuming that $\mu < 1$) yields $W = [I - \beta_e A\Pi]^{-1} rk$. It follows that $W_s = r(\theta_s)k + \beta_e A \sum_{i=1}^{n} W_i \pi_{si}$ and thus that $W_s = V(\theta_s)$.

ii) Let π_s denote the s^{th} row of the transition matrix Π. Then, the first constraint in ($A1$) can be rewritten as

$$\begin{aligned} & \ln(wh) + B(1-\eta)^{-1}(1-h)^{1-\eta} + \beta_w \delta \pi_s U^C + \beta_w (1-\delta) \pi_s U^U \geq U \\ & \ln(wh) + B(1-\eta)^{-1}(1-h)^{1-\eta} + \beta_w \delta \pi_s U^C \\ & \qquad + \beta_w (1-\delta) \pi_s [I - \beta_w \delta \Pi][I - \beta_w \delta \Pi]^{-1} U^U \geq U \\ & \ln(wh) + B(1-\eta)^{-1}(1-h)^{1-\eta} + \beta_w \delta \pi_s U^C + \pi_s [I - \beta_w \delta \Pi] Z U^U \geq U, \end{aligned} \tag{A3}$$

in which $Z = \beta_w (1-\delta)[I - \beta_w \delta \Pi]^{-1}$. Thus, rearranging ($A3$), one obtains

$$\ln(wh) + B(1-\eta)^{-1}(1-h)^{1-\eta} + \beta_w \delta \sum_{i=1}^{n} [U_i^C - \pi_i Z U^U] \pi_{si} \geq U - \pi_s Z U^U$$

or, letting $\tilde{U}^C = U^C - \Pi Z U^U$,

$$\ln(wh) + B(1-\eta)^{-1}(1-h)^{1-\eta} + \beta_w \delta \sum_{i=1}^{n} \tilde{U}_i^C \pi_{si} \geq U - \pi_s Z U^U. \tag{A4}$$

iii) Let $\tilde{U}^H = U^H - \Pi Z U^U$ and let $\tilde{U}^U = U^U - \Pi Z U^U = [I - \Pi Z]U^U$. After substituting $U^H = \tilde{U}^H + \Pi Z U^U$ in the expression for U^U in ($A1$) and solving for U^U, one obtains

$$U^U = [I - \beta_w(1-\mu)\Pi - \mu \Pi Z]^{-1}[(1-\mu)u^U + \mu \tilde{U}^H].$$

Thus,

$$\tilde{U}^U = [I - \Pi Z][I - \beta_w(1-\mu)\Pi - \mu \Pi Z]^{-1}[(1-\mu)u^U + \mu \tilde{U}^H]. \tag{A5}$$

Define a function f from F as $f_s(\tilde{U}) = F_s(\tilde{U} + \pi_s Z U^U) - V(\theta_s)$. Using ($A4$), ($A5$), the fact that $V(\theta_s) = r(\theta_s)k + \beta_e A \sum_{i=1}^{n} W_i \pi_{si}$, and that $W_i = V(\theta_i)$, one has that

$$\begin{aligned} f_s(\tilde{U}) = & \max_{h,w,\tilde{U}^C} \left\{ \theta_s h^\alpha k^{1-\alpha} - wh + \beta_e A \delta \sum_{i=1}^{n} f_i(\tilde{U}_i^C)\pi_{si} \right\} \\ & \text{s.t. } \ln(wh) + B(1-\eta)^{-1}(1-h)^{1-\eta} + \beta_w \delta \sum_{i=1}^{n} \tilde{U}_i^C \pi_{si} \geq \tilde{U} \\ & \quad \tilde{U}^U = [I - \Pi Z][I - \beta_w(1-\mu)\Pi - \mu \Pi Z]^{-1}[(1-\mu)u^U + \mu \tilde{U}^H], \\ & \quad \tilde{U}_i^C \geq \tilde{U}_i^U, \quad i = 1, 2, \ldots, n, \\ & \quad f_i(\tilde{U}_i^C) \geq 0, \quad i = 1, 2, \ldots, n, \end{aligned} \tag{A5}$$

for a vector $\tilde{U}^H$ such that $f_s(\tilde{U}_s^H) = 0$ for all s. Note that in ($A5$), $\tilde{U}^H$ affects the vector $\tilde{U}^U$ uniquely.

Given a vector $\tilde{U}^H$, this dynamic programming problem is essentially similar to that of Thomas and Worrall (1994). Straightforward application of Lemma 1, Lemma 2, and Lemma 4 (Case 2) of the Appendix in Thomas and Worrall (1994) implies that f is strictly decreasing and strictly concave on $(\tilde{U}^U, \tilde{U}^H)$. Essentially, what Thomas and Worrall (1994) show is that the value function may be found as the pointwise limit of an iterative scheme initialized with the function f^* solving a problem similar to ($A5$) in which all but the first constraint are eliminated (i.e. the function arising from the problem under full commitment). The vector $\tilde{U}^H$ is found as follows: First, its initial value is set to the value of U such that $f^*(U) = 0$. Since $f \leq f^*$, this initial value is the smallest upper bound for $\tilde{U}^H$. Then, problem ($A5$) is solved and the resulting value function is used to find a new estimate of $\tilde{U}^H$, which can then be used to solve ($A5$) once again. This process is repeated until the convergence of $\tilde{U}^H$.

Appendix B.

Data for the U.S. were obtained from the Bureau of Labor Statistics (http://stats.bls.gov) Major Sector Productivity Index and Cost Index database (Quarterly Labor Productivity).

- · Output (Y): series ID PRS85006013.
- · Hours (H): series ID PRS85006033.
- · Employment (E): series ID PRS85006043.
- · Real Hourly Compensation (W): series ID PRS85006153.
- · Output per worker: Y/E.
- · Earnings per worker: $W \times H$.
- · Hours per worker: H/E.
- · Average labour product: Y/H.
- · Real wage: W.
- · Labour share: series ID PRS85006173.

All series are seasonally adjusted and pertain to the nonfarm business sector.

Table 1. Selected moments - baseline model.[a]

A. Standard deviations relative to that of output per worker[b]

Series	U.S. data[c]	Contract with no commitment	Contract with full commitment	Spot market
Output per worker	1.38	1.16	1.39	0.95
Earnings per worker	0.73	0.57	0.08	1.00
Hours per worker	0.34	0.34	0.49	0.00
Average labour product	0.81	0.74	0.51	1.00
Real wage	0.63	0.59	0.48	1.00
Labour share	0.76	0.69	0.98	0.00

B. Correlations with output per worker

Output per worker	1.00	1.00	1.00	1.00
Earnings per worker	0.62	0.74	0.29	1.00
Hours per worker	0.68	0.83	0.99	-
Average labour product	0.95	0.97	0.99	1.00
Real wage	0.34	0.22	-0.97	1.00
Labour share	-0.71	-0.83	-0.99	-

C. Autocorrelations

Output per worker	0.77	0.64	0.68	0.68
Earnings per worker	0.84	0.82	0.95	0.68
Hours per worker	0.78	0.51	0.68	-
Average labour product	0.71	0.72	0.69	0.68
Real wage	0.81	0.76	0.68	0.68
Labour share	0.72	0.51	0.68	-

D. Selected correlations

Hours per worker - apl	0.42	0.66	0.99	-
Hours per worker - wage	0.03	-0.35	-0.99	-
Real wage - apl	0.41	0.46	-0.95	1.00

[a] Based on the log of each series detrended with the Hodrick-Prescott filter ($\lambda = 1600$).

[b] Except for output per worker which is the raw standard deviation multiplied by 100.

[c] Quarterly U.S. nonagricultural business data 1954-1999.

Table 2. Sensitivity of results in Table 1.

A. Standard deviations relative to that of output per worker[b]

Series	U.S. data	$\epsilon = 0.67$	$\epsilon = 1.00$	$\epsilon = 1.50$
Output per worker	1.38	1.15	1.16	1.10
Earnings per worker	0.73	0.48	0.57	0.73
Hours per worker	0.34	0.30	0.34	0.30
Average labour product	0.81	0.74	0.74	0.81
Real wage	0.63	0.49	0.59	0.71
Labour share	0.76	0.76	0.69	0.50

B. Correlations with output per worker

Output per worker	1.00	1.00	1.00	1.00
Earnings per worker	0.62	0.69	0.74	0.88
Hours per worker	0.68	0.89	0.83	0.70
Average labour product	0.95	0.98	0.97	0.97
Real wage	0.34	0.11	0.22	0.60
Labour share	-0.71	-0.89	-0.83	-0.70

C. Autocorrelations

Output per worker	0.77	0.66	0.64	0.63
Earnings per worker	0.84	0.83	0.82	0.77
Hours per worker	0.78	0.58	0.51	0.40
Average labour product	0.71	0.71	0.72	0.72
Real wage	0.81	0.78	0.76	0.75
Labour share	0.72	0.58	0.51	0.40

D. Selected correlations

Hours per worker - apl	0.42	0.50	0.66	0.79
Hours per worker - wage	0.03	-0.35	-0.35	-0.13
Real wage - apl	0.41	0.28	0.46	0.78

[a]Except for output per worker which is the raw standard deviation multiplied by 100.

Table 3. Estimates of the intertemporal labour supply elasticity ϵ.[a]

A. Specification: $\Delta\ln(h_{it}) = \alpha_0 + \alpha_1 \Delta\ln(W_{it}) + \varepsilon_{it}$

Results	$\epsilon = 2/3$	$\epsilon = 1$	$\epsilon = 3/2$
$\hat{\alpha}_1$	-0.14 (0.15)	-0.04 (0.20)	0.10 (0.20)
R^2	0.23 (0.19)	0.13 (0.17)	0.05 (0.11)
% times signif. > 0	10	38	74

B. Specification: $\Delta\ln(h_{it}) = \alpha_0 + \alpha_1 \Delta\ln(W_{it} h_{it}) + \varepsilon_{it}$

Results	$\epsilon = 2/3$	$\epsilon = 1$	$\epsilon = 3/2$
$\hat{\alpha}_1/(1-\hat{\alpha}_1)$	0.24 (0.47)	0.26 (0.36)	0.15 (0.17)
R^2	0.02 (0.06)	0.05 (0.08)	0.12 (0.16)
% times signif. > 0	85	81	88

[a]Based on a simulated panel of 10 yearly observations on 2000 workers.
Panel A: IV estimation with instruments: point-in-time hourly wage (Altonji 1986)
Panel B: IV estimation with instruments: time trend, job tenure, and year dummy variables (MaCurdy 1981)
Estimates reported are average estimates over 100 simulations.
Standard deviations are reported within parenthesis.